Resonance Transition of the Spatial Correlation Factor of Self-Generated Oscillations in the Postbreakdown Regime of p-Ge

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The linear correlation factor of spatially coupled nonlinear self-generated oscillations in the post-breakdown regime of p-Ge at 4.2 K is investigated. The observed results can be consistently explained in terms of a two-cell model of energy relaxation oscillations, which yields resonance transitions between correlated and anticorrelated oscillations. These are due to node-focus transitions of the fixed point of the passive cell which is slaved by the active cell.

The nonlinear spatio-temporal dynamics associated with the excitation of different spatial degrees of freedom in nonlinear dynamic systems is of great current interest. In semiconductors a variety of such phenomena has recently been studied both theoretically and experimentally in the regime of low-temperature impurity breakdown, for reviews see [1–3]. In particular, spatial correlations and crosstalk of self-generated oscillations in two electrically separated parts of a single p-Ge crystal have been investigated [4–6].

As explanation a microscopic physical model has been proposed [6] which is based upon impact ionization of shallow acceptors [7] and nonlinear energy relaxation of hot carriers [8, 9]. The crystal is represented by two spatially homogeneous identical cells which are coupled via energy exchange of the hot carriers due to the rapid propagation of hot phonons. In each of the cells self-generated energy relaxation oscillations can arise. If two different voltages are applied to the two cells, the model yields complex interactions of the oscillations, including quasiperiodicity and mode-locking which obeys the Farey tree ordering [10, 11].

In this paper we present theoretical and experimental investigations of p-Ge at 4.2 K which show that a sharp resonance transition between a state where the two subsystems oscillate in phase, and a state where they oscillate with a phase-lag of π, can be induced by varying the bias applied to one cell, and holding the bias at the other cell fixed. The results obtained from the microscopic two-cell model are in good agreement with our experimental findings.

In Fig. 1 the experimental configuration is shown. As described elsewhere [5] in detail, the semiconductor sample was prepared from single-crystalline p-Ge with an acceptor doping concentration of about 10^{14} cm^{-3} and typical dimensions of about $0.25 \times 2 \times 8$ mm^3. Four ohmic Al contacts (hatched areas) evaporated onto one of the two largest surfaces divide the Ge crystal into three separate subsections. To probe the spatial correlation between different parts of the same crystal, the bias voltages V_1 and V_2 were applied to the two outer subsystems 1 and 2, respectively, while the potential difference between the inner contact electrodes was always kept zero. In this way, no charge carriers could be transported across the intermediate subsection, such that lattice heat conductivity remained as the only important coupling mechanism in between. Our experimental system then consisted of two electrically separated and diffusively coupled subsections, each of which could electrically be driven into impact ionization breakdown capable of generating spontaneous current oscillations. The resulting sample currents I_1 and I_2 were measured using load resistances (1 Ω) connected in series to the correspond-

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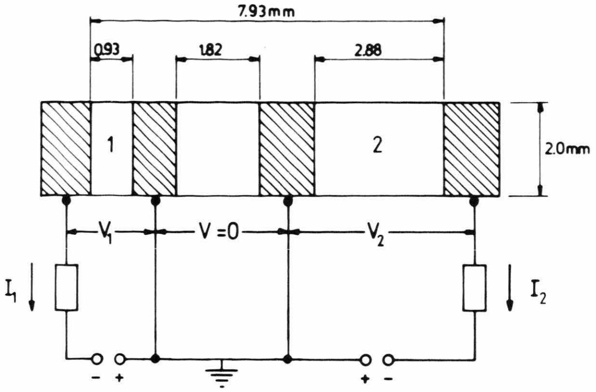

Fig. 1. Scheme of the experimental set-up.

ing subsections 1 and 2. During the experiments, the sample investigated was kept at the liquid-helium temperature of 4.2 K and carefully shielded against external irradiation (visible and far infrared).

The model is given by the following set of balance equations [9] for the hole concentrations p_i and the mean energies of the holes E_i in the two cells ($i = 1, 2$):

$$\dot{p}_1 = f_p(p_1, E_1), \tag{1}$$

$$\dot{E}_1 = f_E(p_1, E_1) + D(E_2 - E_1), \tag{2}$$

$$\dot{p}_2 = f_p(p_2, E_2), \tag{3}$$

$$\dot{E}_2 = f_E(p_2, E_2) + D(E_1 - E_2) \tag{4}$$

with

$$f_p(p, E) = X(E)\, p\, (N_A^* - p) - T^s(E)\, [p\, (N_D + p) - p_0\, (N_A^* - p)], \tag{5}$$

$$f_E(p, E) = e\, \mu\, \mathcal{E}_i^2 - (E - E_0)/\tau_e - E_{\text{th}}\, X(E)\, (N_A^* - p) - E f_p(p, E)/p, \tag{6}$$

where the impact ionization coefficient X and the capture coefficient T^s are modelled by the phenomenological functions

$$X(E) = X_0 \exp(-E_{\text{th}}/E)\, (E/E_{\text{th}})^{-r}, \tag{7}$$

$$T^s(E) = \begin{cases} T_0^s & E < E_r \\ T_0^s\, [(s+1) - s\, (E_r/E)]\, (E/E_r)^{-s}, & E \geqq E_r \end{cases} \tag{8}$$

with positive parameters X_0, E_{th}, r, T_0^s, E_r, s. $N_A^* \equiv N_A - N_D$, N_A, and N_D are effective doping concentration, acceptor and compensating donor density, respectively, μ is the hole mobility, $\mathcal{E}_i$ the electric field applied to cell i, E_0 the electron energy in thermal equilibrium with the lattice, τ_e the energy relaxation time, E_{th} the impact ionization threshold energy, and p_0 the thermal equilibrium hole concentration for a Fermi level coinciding with the acceptor level. The p_0-term in (5) describes thermal ionization of holes [1]. As convenient control parameters we introduce the dimension-

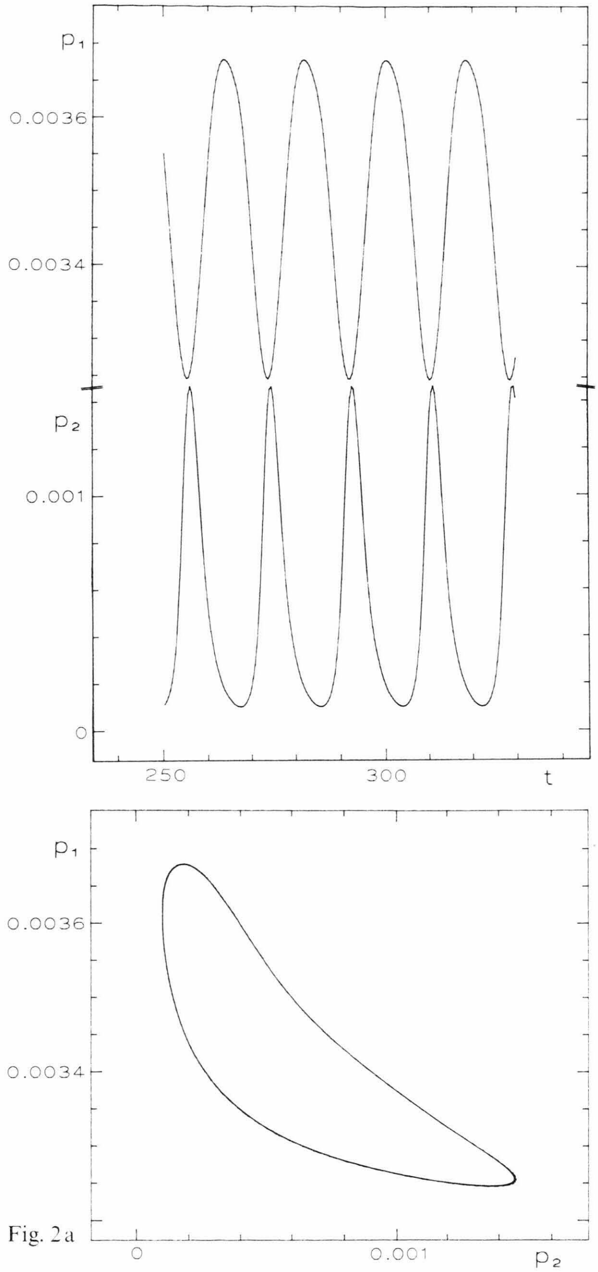

Fig. 2a

Fig. 2. Calculated carrier densities p_1 and p_2 in units of N_A^* versus time t in units of τ_e, and phase portraits of p_1 versus p_2 for $P_2 = 11$, and (a) $P_1 = 5$, (b) $P_1 = 8$, (c) $P_1 = 10$, (d) $P_1 = 14$, (e) $P_1 = 16$. The transients have been discarded.

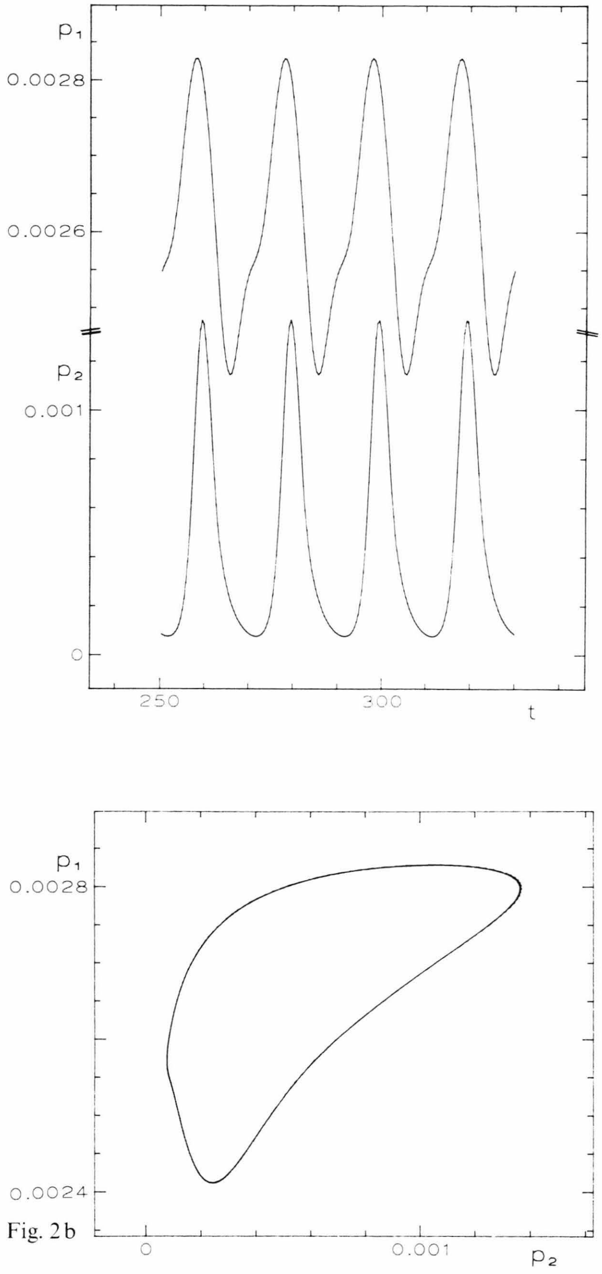

Fig. 2b

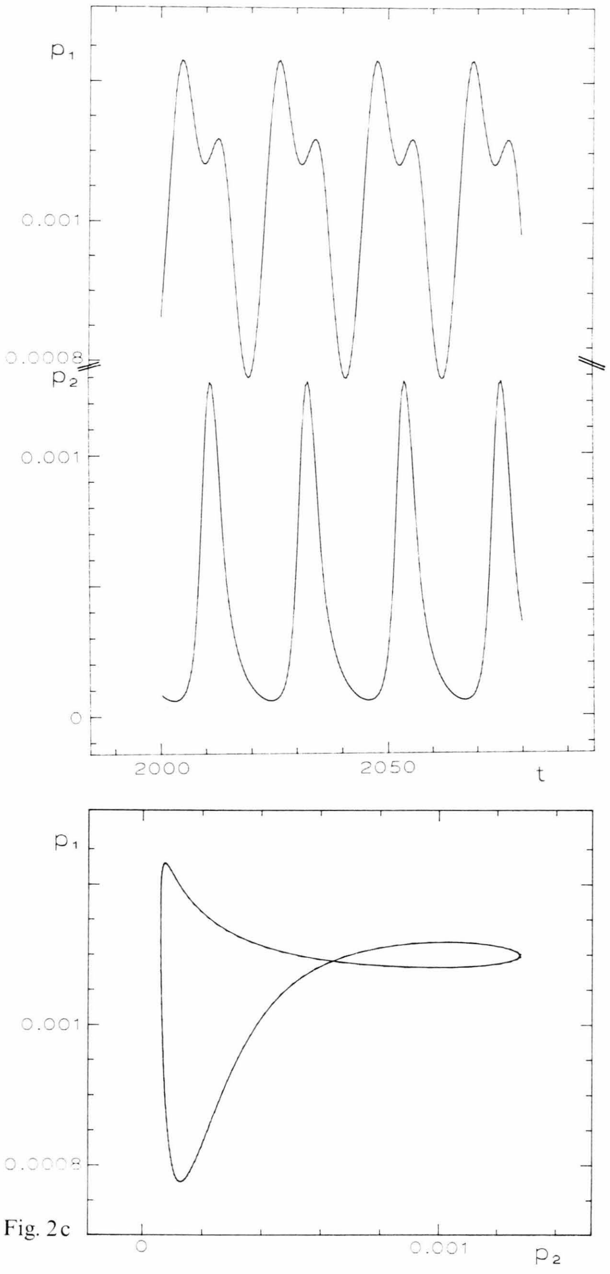

Fig. 2c

less net absorbed power per carrier in each cell:

$$P_i=(\mathrm{e}\,\mu\,\mathcal{E}_i^2\,\tau_\mathrm{e}+E_0)/E_\mathrm{th}, \qquad i=1,2. \tag{9}$$

The material parameters used in the following are $N_\mathrm{A}=10^{14}\,\mathrm{cm}^{-3}$, $N_\mathrm{D}=10^{12}\,\mathrm{cm}^{-3}$, $p_0=3\times10^{-11}\,N_\mathrm{A}^*$, $\tau_\mathrm{e}=10^{-7}\,\mathrm{s}$, $E_\mathrm{th}=10\,\mathrm{meV}$, $T_0^s=3\times10^{-5}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$, $X_0=10^{-6}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$, $E_r=10\,E_\mathrm{th}$, $r=0.5$, $s=1.5$, $D=0.05/\tau_\mathrm{e}$, corresponding to p-Ge at 4.2 K [9].

Figure 2 depicts a sequence of time series and phase portraits obtained from (1)–(8) by varying P_1, and holding $P_2=11$ fixed. The time series and phase portraits in Figs. 2 (a), (e) and (b), (d) reveal an approximately linear anticorrelation or correlation between p_1 and p_2, i.e., the two cells oscillate (a), (e) with a phase-lag of π, and (b), (d) in phase. Such behaviour was indeed observed (Figure 3). A quantitative mea-

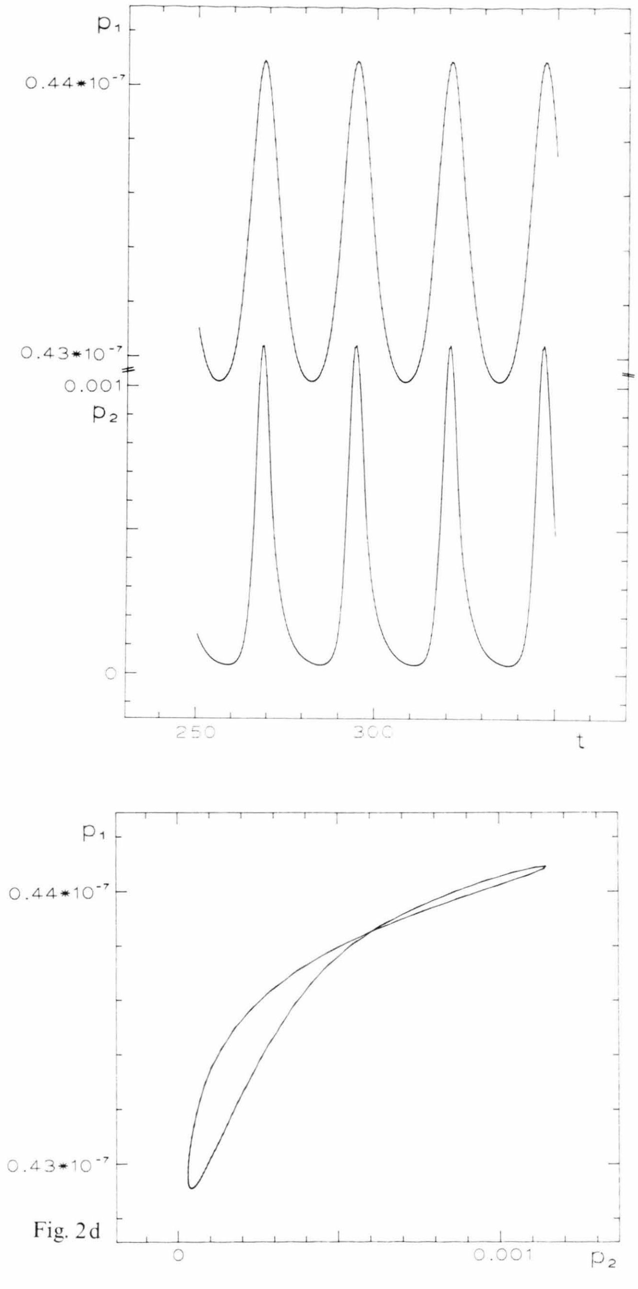

Fig. 2d

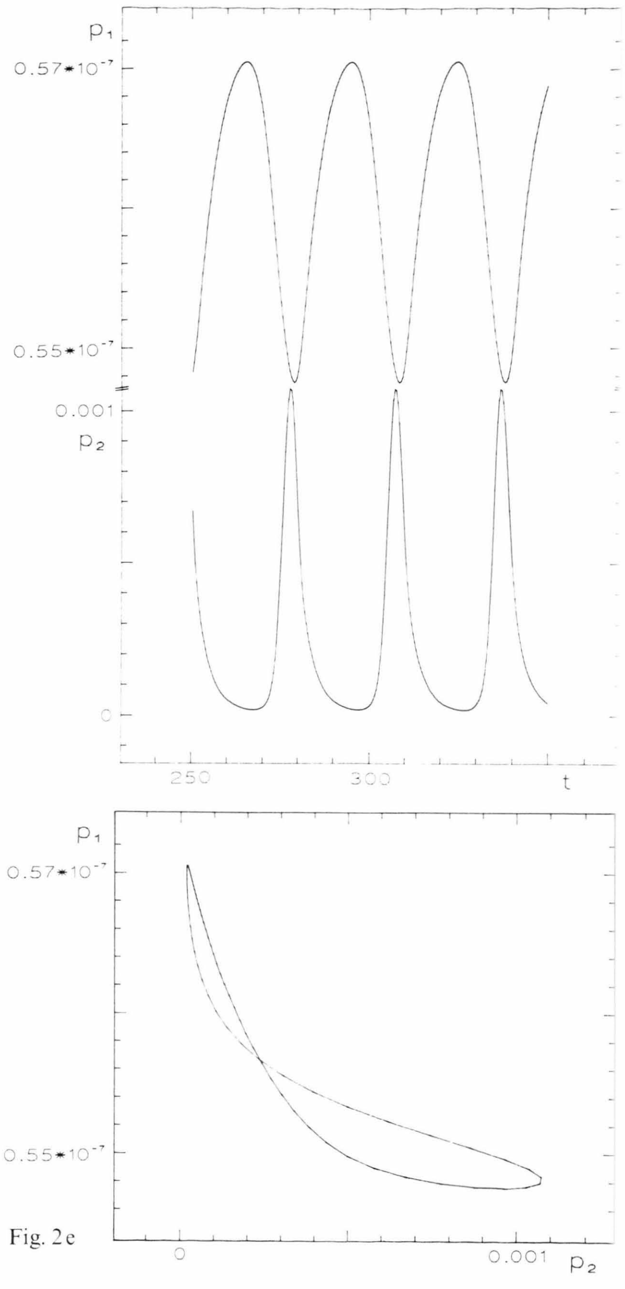

Fig. 2e

sure of this *linear* correlation is given by the linear correlation factor

$$F = \frac{\langle \delta p_1(t)\, \delta p_2(t) \rangle}{[\langle \delta p_1(t)^2 \rangle \langle \delta p_2(t)^2 \rangle]^{1/2}} \tag{10}$$

with $\delta p_i = p_i - \langle p_i \rangle$, $\langle p_i \rangle = \lim\limits_{T \to \infty} \frac{1}{T} \int\limits_0^T \mathrm{d}t\, p_i(t)$.

The dependence of F upon P_1 shows five different regimes: (i) anticorrelation of p_1 and p_2 ($F \simeq -1$) for $0 \lesssim P_1 \lesssim 6$, (ii) correlation ($F \simeq +1$) for $6 \lesssim P_1 \lesssim 10$, (iii) no linear correlation ($F \simeq 0$) for $10 \lesssim P_1 \lesssim 13$ (except for the symmetric case $P_1 = P_2 = 11$), (iv) correlation for $13 \lesssim P_1 \lesssim 14$, and (v) anticorrelation for $14 \lesssim P_1 \lesssim 17$. Between regimes (i) and (ii), or (iv) and (v), there occur

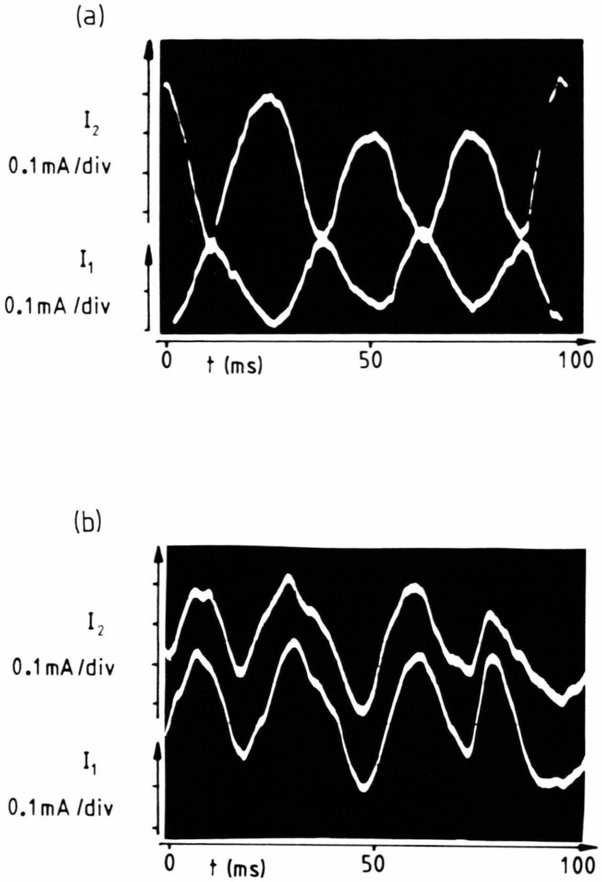

Fig. 3. Observed time series of the currents I_1 and I_2 in the two sample subsystems for (a) $V_1 = 3.40$ V, $V_2 = 2.60$ V; (b) $V_1 = 1.80$ V, $V_2 = 1.06$ V.

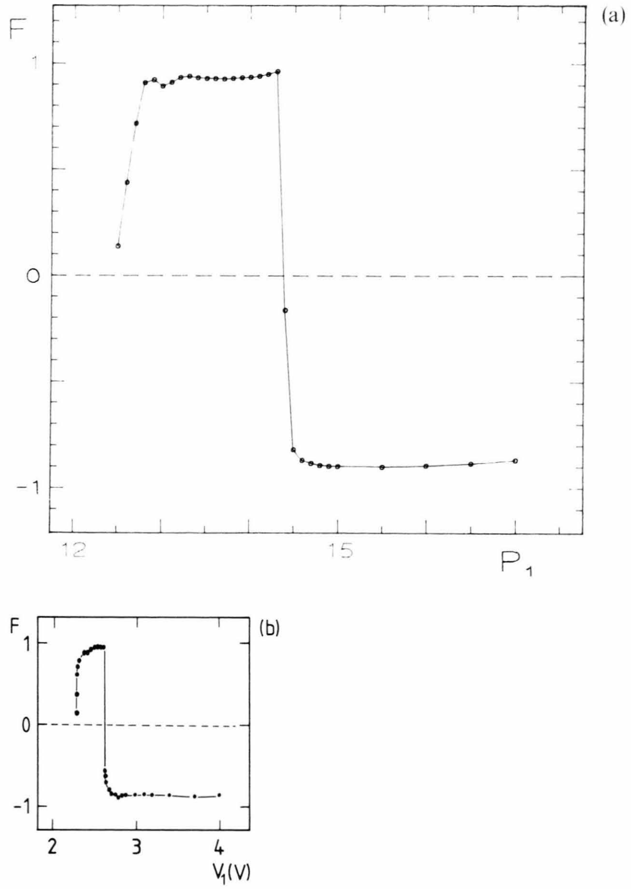

Fig. 4. Linear correlation factor F as a function of the applied bias in subsystem 1.
(a) Theoretically with $P_2 = 11$,
(b) experimentally with $V_2 = 1.0$ V.

very sharp transitions. Experimentally, the linear correlation factor between the currents I_1 and I_2 has been determined as a function of the voltage V_1 applied to subsystem 1 for fixed bias voltage V_2. Figure 4 shows that the agreement between theory and experiment is excellent.

The discontinuous behaviour of the linear correlation factor can be physically understood by noting that the five regimes (i)–(v) correspond to five qualitatively different states of the dynamic system (1)–(4). A linear stability analysis of its fixed point reveals that it undergoes a characteristic sequence of transitions [10, 11]. While P_2 is fixed in a range where the subsystem 2 would be – without coupling ($D = 0$) – in an actively oscillating state (stable limit cycle), P_1 is varied in a range where – without coupling – the subsystem 1 would successively display (i) a stable node, (ii) a stable focus, (iii) a stable limit cycle, (iv) a stable focus, (v) a stable node. The coupling does not essentially affect this behaviour, so that we have two "active" subsystems in the regime (iii), and one "active" and one "passive" subsystem elsewhere. In the regime (iii) the linear correlations break down ($F \simeq 0$), and complex mode-locking structures and quasiperiodicity occur [11]. In the other regimes the passive subsystem 1 is slaved by the active system 2, and linear (anti-) correlations occur. In the regimes (i) and (v) the passive subsystem behaves as an overdamped oscillator (stable node), and follows the active subsystem with phase-reversal ($F \simeq -1$). In the regimes (ii) and (iv) the passive subsystem is a damped oscillator (stable focus) with an intrinsic frequency mostly larger than that of the active system, and oscillates in phase ($F \simeq 1$), like a periodically driven, damped linear oscillator below resonance.

Thus the sharp transitions between anticorrelation ($F = -1$) and correlation ($F = +1$) represent a resonance phenomenon associated with a node-focus transition of the passive subsystem. Finally, it should be pointed out that the linear correlation factor F becomes meaningless if the oscillations in the two subsystems have a phase lag other than π or zero, or if they are *nonlinearly* correlated, for instance, in the regime (iii) of two active oscillators. A characteristic nonlinear measure, which is particularly sensitive to correlations, and which is meaningful both in the active and in the passive regimes, is the bit-number variance of the asymptotic invariant density of the dynamic system [12], which was originally introduced in nonequilibrium thermodynamics as a generalization of the specific heat [13]. An experimental investigation of this quantity is currently in preparation.

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